MIDTERM EXAM 1 (Practice Exam)

MATH 312, GROUP 001

Version A

Name:

The use of calculators, computers and similar devices is neither necessary nor permitted during this exam. Correct answers without proper justification will not receive full credit. Clearly highlight your answers and the steps taken to arrive at them: illegible work will not be graded. You may use both sides of one 8×11 cheat-sheet.

Problem Number	Possible Points	Points Earned
1	25	
2	20	
3	20	
4	15	
5	20	
Total	100	

Problem 1

[25 points] Consider the matrix $A = \begin{bmatrix} 1 & 3 & 1 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 \\ 2 & 6 & 4 & 4 & 3 \\ 0 & 0 & 2 & 2 & 3 \end{bmatrix}$.

Part a. [5 points] Find the matrix R, the RREF (row reduced echelon form) of A.

Part b. [6 points] Find the matrix E that transforms A into R, i.e., $EA = R$.

Part c. [6 points] Find the complete solution to $A\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}$.

Part d. [8 points] Find a basis for each one of the four fundamental subspaces of A.

Problem 2

[20 points] A is a 2 by 4 matrix with nullspace given by all vectors of the form

$$\alpha \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 6 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \qquad \alpha, \beta \in \mathbb{R}.$$

Part a. [4 points] The linear application given by A goes from $\mathbb{R}^4$ to $\mathbb{R}^m$. Which is the value of m?

Part a. [4 points] What is the dimension of $N(A)$?

Part a. [6 points] What is the column space of A?

Part a. [6 points] How many pivot columns are there? Find the reduced row echelon form R of A.

Problem 3

[20 points] Let $A = \begin{bmatrix} 1 & 2 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 & 3 & 1 \end{bmatrix}$.

Part a. [10 points] Find a basis for $C(A)$.

Part b. [10 points] Show that $\mathbf{b} = \begin{bmatrix} 2 & 0 & 1 & 8 \end{bmatrix}^T$ is in the column space of A by writing $\mathbf{b}$ as a linear combination of the vectors in the basis of $C(A)$.

Problem 4

[15 points] Consider the plane $2x - y + z = 0$ in $\mathbb{R}^3$.

Part a. [7 points] Find a basis for that plane.

Part b. [8 points] Find the projection matrix onto that plane.

Problem 5

[20 points: 2.5 points each] In each of the following cases, clearly mark the statement as **true** or **false**. Please also explain your answers in order to receive credit for this problem.

1. If A and B have inverses A^{-1} and B^{-1}, the $(A+B)^{-1} = B^{-1} + A^{-1}$.
2. An square matrix A such that $N(A) = \{0\}$ always has an inverse.
3. Consider a matrix that adds row 1 to row 2 and at the same time adds row 2 to row 1. Then, the inverse matrix subtracts row 1 from row 2 and at the same time subtracts row 2 from row 1.
4. In $\mathbb{R}^3$, the nullspace of the projection onto a line is another line perpendicular to it.
5. If AB is invertible, then A and B are invertible.
6. If a 3×5 matrix has RREF with three pivots, then its rows form a basis of $\mathbb{R}^5$.
7. The set of points satisfying $y = 3x^2$ is a subspace of $\mathbb{R}^2$.
8. Let V be the vector space of polynomials with degree no more than 3. The set of polynomials of the form $p(x) = ax^2$ with $a \in \mathbb{R}$ is a subspace of V.