HOMEWORK ASSIGNMENT 5

Name: **Due:** Friday October 12

Problem 1

Given the input data $x = 0, 1, 1, 2$, $y = 0, 1, 1, 3$ and the output data $z = 0, 1, 2, 2$,

1. Find the plane that best fits the data.
2. Find the paraboloid $z = ax^2 + by^2 + c$ that best fits the data.

Problem 2: Strang 4.3 #12, page 230

This problem projects $\mathbf{b} = (b_1, \dots, b_m)$ onto the line through $\mathbf{a} = (1, \dots, 1)$. We solve m equations $\mathbf{a}x = \mathbf{b}$ in 1 unknown (by least squares).

1. Solve $\mathbf{a}^T\mathbf{a}\hat{x} = \mathbf{a}^T\mathbf{b}$ to show that $\hat{x}$ is the *mean* (the average) of the $\mathbf{b}$'s.
2. Find $\mathbf{e} = \mathbf{b} - \mathbf{a}\hat{x}$ and the *variance* $\|\mathbf{e}\|^2$ and the *standard deviation* $\|\mathbf{e}\|$.
3. The horizontal line $\hat{\mathbf{b}} = 3$ is closest to $\mathbf{b} = (1, 2, 6)$. Check that $\mathbf{p} = (3, 3, 3)$ is perpendicular to $\mathbf{e}$ and find the 3 by 3 projection matrix P.

Problem 3: Strang 4.4 #11, page 243

1. Gram-Schmidt: Find orthonormal vectors $\mathbf{q}_1$ and $\mathbf{q}_2$ in the plane spanned by $\mathbf{a} = (1, 3, 4, 5, 7)$ and $\mathbf{b} = (-6, 6, 8, 0, 8)$.
2. Which vector in this plane is closest to $(1, 0, 0, 0, 0)$?

Problem 4

1. Find an orthonormal basis (using Gram-Schmidt method) for the subspace S in $\mathbb{R}^4$ spanned by all solutions of
$$x_1 + x_2 + x_3 - x_4 = 0.$$
2. Find an orthonormal basis for the orthogonal complement $S^\perp$.
3. Find $\mathbf{b}_1$ in S and $\mathbf{b}_2$ in $S^\perp$ so that $\mathbf{b}_1 + \mathbf{b}_2 = (1, 1, 1, 1)$.

Problem 5: Section 8.1 #4, page 408

If S and T are linear transformations, is $T(S(\mathbf{v}))$ linear or quadratic?

1. (Special case) If $S(\mathbf{v}) = \mathbf{v}$ and $T(\mathbf{v}) = \mathbf{v}$, then $T(S(\mathbf{v})) = \mathbf{v}$ or $\mathbf{v}^2$?
2. (General case) $S(\mathbf{v}_1+\mathbf{v}_2) = S(\mathbf{v}_1)+S(\mathbf{v}_2)$ and $T(\mathbf{v}_1+\mathbf{v}_2) = T(\mathbf{v}_1)+T(\mathbf{v}_2)$ combine into
$$T(S(\mathbf{v}_1 + \mathbf{v}_2)) = T(\underline{\qquad\quad}) = \underline{\qquad\quad} + \underline{\qquad\quad}$$

Problem 6: Section 8.1 #6, page 408

Which of these transformations satisfy $T(\mathbf{v} + \mathbf{w}) = T(\mathbf{v}) + T(\mathbf{w})$ and which satisfy $T(c\mathbf{v}) = cT(\mathbf{v})$? Which are linear transformations?

1. $T(\mathbf{v}) = \mathbf{v}/\|\mathbf{v}\|$
2. $T(\mathbf{v}) = v_1 + v_2 + v_3$
3. $T(\mathbf{v}) = (v_1, 2v_2, 3v_3)$
4. $T(\mathbf{v}) =$ largest component of $\mathbf{v}$.

Problem 7: Strang 8.1 #29, page 408

What conditions on $\det A = ad - bc$ ensure that the output house AH (see book) will

1. be squashed onto a line?
2. keep its endpoints in clockwise order (not reflected)?
3. have the same area as the original house?

Problem 8: Strang 8.2 #1,2, page 418

1. The transformation S takes the *second derivative*. Keep $1, x, x^2, x^3$ as the input basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ and also as output basis $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4$. Write $S(\mathbf{v}_1), S(\mathbf{v}_2), S(\mathbf{v}_3), S(\mathbf{v}_4)$ in terms of the $\mathbf{w}$'s. Find the 4 by 4 matrix A_2 for S.
2. What functions have $S(\mathbf{v}) = \mathbf{0}$? They are in the kernel of the second derivative S. What vectors are in the nullspace of its matrix A_2?

Problem 9: Strang 8.2 #15, page 419

1. What matrix M transforms $(1, 0)$ and $(0, 1)$ to (r, t) and (s, u)?
2. What matrix N transforms (a, c) and (b, d) to $(1, 0)$ and $(0, 1)$?
3. What condition on a, b, c, d will make part (b) impossible?

Problem 10:

Read Chapter 5 (determinants). Which concept was most difficult to you?