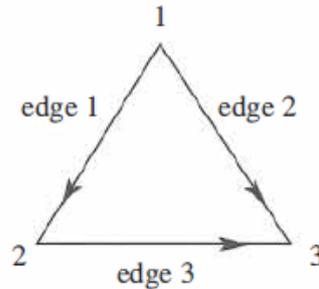# HOMEWORK ASSIGNMENT 10

Name: **Due:** Tuesday December 4 4pm

PROBLEM 1: STRANG 10.1 #1, PAGE 459

Write down the 3 by 3 incidence matrix A for the triangle graph. The first row has -1 in column 1 and +1 in column 2. What vectors (x_1, x_2, x_3) are in its nullspace? How do you know that $(1, 0, 0)$ is not in its row space?

PROBLEM 2: STRANG 10.1 #2, PAGE 459

Write down A^T for the triangle graph. Find a vector $\mathbf{y}$ in its nullspace. The components of $\mathbf{y}$ are currents on the edges - how much current is going around the triangle?

PROBLEM 3: STRANG 10.1 #5, PAGE 460

For the triangle graph above, choose a vector (f_1, f_2, f_3) for which $A^T\mathbf{y} = \mathbf{f}$ can be solved, and a vector $\mathbf{f}$ that allows no solution. How are those $\mathbf{f}$'s related to $\mathbf{x} = (1, 1, 1)$? The equation $A^T\mathbf{y} = \mathbf{f}$ is Kirchhoff's _____ law.

PROBLEM 4: STRANG 10.1 #9, PAGE 460

For the square graph below, find two requirements on the b's for the five differences $x_2 - x_1$, $x_3 - x_1$, $x_3 - x_2$, $x_4 - x_2$, $x_4 - x_3$ to equal b_1, b_2, b_3, b_4, b_5. You have found Kirchhoff's _____ law around the two _____ in the graph.
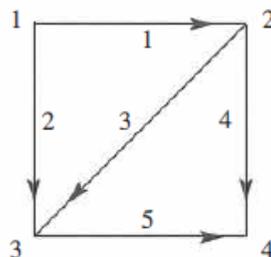
PROBLEM 5

The following matrix is an incidence matrix of some graph:

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

a) How many nodes must this graph have? How many edges?

b) Using only the matrix A, find the number of loops in this graph, and find the number of connected components.

c) Draw a graph with A as its incidence matrix and verify your answers to the previous part of this problem.
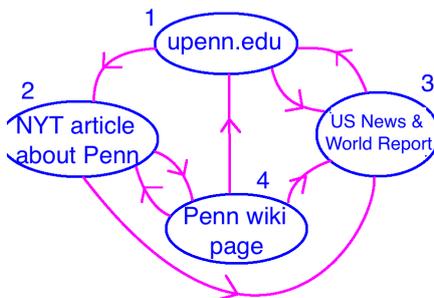
PROBLEM 6

The following matrix is the adjacency matrix of some graph:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

a) How many nodes must this graph have? What are the degrees of each node?

b) Using only the matrix A, find the number of paths of length 1 from node 1 to node 4, and the number of paths of length 2 from node 1 to node 4.

c) Draw a graph with A as its adjacency matrix and verify your answers to the previous part of this problem.

PROBLEM 7

a) Write the transition matrix for this graph, adhering to the numbering of nodes shown in the image.

b) Find the PageRank.

Problem 8

Read section 10.4 of Strang. What was the most difficult for you?