HOMEWORK ASSIGNMENT

Name: **Due:** Wednesday March 13, 4PM

Problem 1:

Let V be the vector space of polynomials of degree less or equal than 1, with basis $\{1, x-2\}$. Let U be the vector space of polynomials of degree less than or equal to 2, with basis $\{1, x, x^2+x\}$. Let $T : V \to U$ be the linear transformation which sends a polynomial p to the polynomial $p \cdot (x+1)$.

Find the matrix of the linear transformation T (multiplication by $x+1$), using the bases for V and U as given above.

Problem 2:

If S is the subspace of $\mathbb{R}^3$ containing only the zero vector, what is $S^\perp$? If S is spanned by $(1,1,1)$, what is $S^\perp$? If S is spanned by $(1,1,1)$ and $(1,1,-1)$, what is a basis for $S^\perp$?

Problem 3:

Construct a matrix with the required property or say why that is impossible:

1. $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a solution and $A^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
2. Every row is orthogonal to every column (A is not the zero matrix)
3. Columns add up to a column of zeros, rows add to a row of 1's.

Problem 4:

Given the input data $x = 0, 1, 1, 2$, $y = 0, 1, 1, 3$ and the output data $z = 0, 1, 2, 2$,

1. Find the plane that best fits the data.
2. Find the paraboloid $z = ax^2 + by^2 + c$ that best fits the data.

Problem 5:

Consider a square matrix A with columns of A denoted by $a_1, a_2, \ldots, a_n$, which satisfies the property:

$$\vec{a_i} \cdot \vec{a_i} = 1 \quad \text{for } i = 1, 2, \ldots, n,$$
$$\vec{a_i} \cdot \vec{a_j} = 0 \quad \text{for } i \neq j,$$

that is, different columns are perpendicular and all columns have norm 1. The same happens to the rows. These matrices are called *orthogonal*.

Calculate the matrices A^TA and AA^T.

Problem 6: Challenge Problems from the zyBook

Challenge activities 4.1.2, 4.5.1, 4.6.1, 5.3.1, 5.3.2 and 5.3.3 of the zyBook. These are not optional.

Problem 7:

Read Chapter 5 (Orthogonality) and Chapter 6 (Determinants) from the zyBook and do all of the participation exercises therein. Which concept was most confusing for you?