HOMEWORK ASSIGNMENT 6

Name: **Due:** Friday March 29, 4PM

All the problems in this homework are from W. Strauss book.

Note: Check you know how to do problems 4, 5, 6 in Section 5.2 (they were done in class).

Problem 1: Strauss, Section 5.2 #8, p.117

(a) Prove that differentiation switches even functions to odd ones, and odd functions to even ones.

(b) Prove the same for integration provided that we ignore the constant of integration.

Problem 2: Strauss, Section 5.2 #9, p.117

Let $\phi(x)$ be a function of period π. If $\phi(x) = \sum_{n=1}^{\infty} a_n \sin(nx)$ for all x, find the odd coefficients (and explain how).

Problem 3: Strauss, Section 5.2 #11, p.118

Find the Fourier series of e^x on $(-L, L)$ in its real and complex forms. (Hint: It is convenient to find the complex form first.)

Problem 4: Strauss, Section 5.2 #15, p.118

Without any computation, predict which of the Fourier coefficients of $|\sin x|$ on the interval $(-\pi, \pi)$ must vanish, and say why.

Problem 5: Strauss, Section 5.3 #2, p.122

(a) On the interval $[-1, 1]$, show that the function x is orthogonal to the constant functions.

(b) Find a quadratic polynomial that is orthogonal to both 1 and x.

(c) Find a cubic polynomial that is orthogonal to all quadratics. (These are the first few *Legendre polynomials*.)

Problem 6: Strauss, Section 5.3 #3, p.123

Consider $u_{tt} = c^2 u_{xx}$ for $0 < x < L$, with the boundary conditions $u(0,t) = 0$, $u_x(L,t) = 0$ and the initial conditions $u(x,0) = x$, $u_t(x,0) = 0$. Find the solution explicitly in series form (including the coefficients).

Problem 7: Strauss, Section 5.3 #6, p.123

Find the complex eigenvalues of the first-derivative operator d/dx subject to the boundary condition $X(0) = X(1)$. Are the eigenfunctions orthogonal on the interval $(0,1)$?

Problem 8: Strauss, Section 5.3 #10, p.123

(*The Gram-Schmidt orthogonalization procedure*) If $X_1, X_2, \dots$ is any sequence (finite or infinite) of linearly independent vectors in any vector space with an inner product, it can be replaced by a sequence of linear combinations that are mutually orthogonal. The idea is that at each step one subtracts off the components parallel to the previous vectors. The procedure is as follows. First, we let $Z_1 = X_1/\|X_1\|$. Second, we define

$$Y_2 = X_2 - (X_2, Z_1)Z_1, \qquad Z_2 = \frac{Y_2}{\|Y_2\|}.$$

Third, we define

$$Y_3 = X_3 - (X_3, Z_1)Z_1 - (X_3, Z_2)Z_2, \qquad Z_3 = \frac{Y_3}{\|Y_3\|},$$

and so on.

(a) Show that all the vectors $Z_1, Z_2, Z_3, \dots$ are orthogonal to each other.

(b) Apply the Gram-Schmidt algorithm to the pair of functions $\cos(x) + \cos(2x)$ and $3\cos(x) - 4\cos(2x)$ in the interval $(0, \pi)$ to get an orthogonal pair.

Note: Check you know how to do problems 11, 12, 13 in Section 5.3 (they were done in class).

Problem 9: Strauss, Section 5.3 #15, p.124

Use the same idea as in Exercise 12 and 13 (of W. Strauss book) to show that none of the eigenvalues of the fourth-order operator d^4/dx^4 with the boundary conditions $X(0) = X(L) = X''(0) = X''(L) = 0$ are negative.