Name ______________________________

Math 312 - Section 002 - Midterm 2 (Practice exam)
Tuesday, April 2, 2019, @ 10:30 AM - 11:50 AM

No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity. I certify that all of the work on this test is my own.

Signature: ______________________

The use of calculators, computers and similar devices is neither necessary nor permitted during this exam. Correct answers without proper justification will not receive full credit. Clearly highlight your answers and the steps taken to arrive at them: illegible work will not be graded. You may use both sides of one 8×11 cheat-sheet.

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Problem	Points	Your score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1 [20 points]

We want to find the curve $y = a + 2^t b$ that gives the best fit (in the least squares sense) to the data $t = 0, 1, 2$, $y = 6, 4, 0$.

Part a.

1. Write down the 3 equations that would be satisfied if the curve went though all 3 points. Then, write the system in matrix form.

2. Find the coefficients a, b of the curve of best fit $y = a + 2^t b$.

Part b. Construct a matrix with the required property or say why that is impossible:

1. $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A and $A^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2. Rows add up to a row of zeros, columns add to a column of 1's.

3. Every row is orthogonal to every column (A is not the zero matrix)

4. A is a diagonalizable matrix and $\lambda = 0$ is one of its eigenvalues.

Problem 2 [20 points]

Part a. Which of the following matrices are diagonalizable? Explain why or why not.

$$A = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 5 \\ 1 & 3 & 5 \end{bmatrix}, \qquad C = \begin{bmatrix} -7 & 13 \\ 13 & 1 \end{bmatrix}$$

Part b. Compute the diagonalization of one of the matrices in Part a. (you can choose which one).

Consider the linear differential system

$$
\begin{aligned}
x' &= 2x - 4y \\
y' &= x - 2y.
\end{aligned}
$$

Part c. For which matrix A can we rewrite this system as $\begin{bmatrix} x \\ y \end{bmatrix}' = A \begin{bmatrix} x \\ y \end{bmatrix}$? Check that this matrix A is non diagonalizable.

Part d. Find the exponential matrix e^{At}. (Hint: Compute first A^2.)

Part e. Find the solution $x(t)$ and $y(t)$ to this linear differential system subject to the initial conditions $x(0) = 3$ and $y(0) = 1$.

Problem 3 [20 points]

Let V be the subspace of vectors in $\mathbb{R}^4$ (with coordinates (w, x, y, z)) such that

$$-w + x - 2z = 0$$

Part a. Find a basis for V.

Part b. Apply Gram–Schmidt to find an orthonormal basis for V.

Part c. Find the closest point in V to $(1, -1, 2, 0)$.

Problem 4 [20 points]

Let $V = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0\}$. Consider the following linear transformation T: projection of vectors in $\mathbb{R}^3$ onto V.

Part a. If P is the usual 3 by 3 projection matrix (i.e., the matrix of the linear transformation T using the standard basis), find three eigenvalues and three independent eigenvectors of P. (Hint: No need to compute P).

Part c. Find an orthonormal basis $\mathcal{V}$ for V.

Part d. Find the matrix of the linear transformation T when the input basis is $\mathcal{U}$,

$$\mathcal{U} = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\} = \{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \},$$

and the output basis is $\mathcal{V}$.

Problem 5 [20 points]

In each of the following cases, clearly mark the statement as **true** or **false**. Please also explain your answers in order to receive credit for this problem.

1. If A is a 3x3 matrix with determinant 1, then $2A$ has determinant 6.

2. If Q is an orthogonal matrix, then the corresponding linear transformation preserves lengths i.e., length of $Q\vec{x}$ is equal to length of $\vec{x}$.

3. The matrix $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$ has an eigenvalue equal to 9.

4. An square matrix with orthonormal columns always has orthonormal rows.

5. The eigenvectors for the zero eigenvalue span the null space of A.

6. Let A be an n by n matrix. If n is odd and A is skew-symmetric (i.e., $A^T = -A$), then A is not invertible.

7. If A is a 2 by 2 matrix with eigenvalues -1 and 2, and B is a 2 by 2 matrix with eigenvalues 0 and 1, then $\det((B - I)^2 A) = 1$.

8. A 2 by 2 real matrix that rotates every vector $90^{\underline{o}}$ cannot have any real eigenvalues.

Extra space for work:

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