HOMEWORK ASSIGNMENT 7

Name: **Due:** Friday April 12, 4pm

Problem 1:

Let $x(t)$ be the number of people at Penn without mumps and $y(t)$ the number of people at Penn with mumps.

We model the populations assuming that more students come to Penn over time (successful recruitment by students without mumps) at a rate of $4x$, and assuming that the population of students without mumps is decreased if there are many students with mumps. Also, assume the mumps population has a fast growth (from $5y$).

So the evolution of this disease among Penn students is modeled by

$$x'(t) = 4x(t) - 2y(t),$$
$$y'(t) = 5y(t).$$

a) Rewrite this as a matrix equation. Under this model, what is the population of people with mumps at time t if initially there are 5 people with mumps and 25,000 people without mumps?

b) Under this model, what is the ratio of infected people versus non infected ones as time goes to infinity, given the initial conditions above?

c) How can you conclude from b) that this is a bad model?
(Note: many models for infections disease are not linear and therefore outside of the purview of this class).

Problem 2:

Write A and B in the form

$$\lambda_1 \vec{x}_1 \vec{x}_1^T + \lambda_2 \vec{x}_2 \vec{x}_2^T$$

of the spectral theorem QDQ^T.

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$$

(keep $||\vec{x}_1|| = ||\vec{x}_2|| = 1$).

Problem 3:

What number b in

$$A = \begin{bmatrix} 2 & b \\ 1 & 0 \end{bmatrix}$$

makes $A = QDQ^T$ possible? What number will make it impossible to diagonalize A? What number will make A singular?

Problem 4:

Give a quick reason why each of these statements is true:

1. Every positive definite matrix is invertible.
2. The only positive definite projection matrix is $P = I$.
3. A diagonal matrix with positive diagonal entries is positive definite.
4. A symmetric matrix with a positive determinant might not be positive definite!

Problem 5: Challenge Problems from the zyBook

Challenge activities 7.5.1, 7.5.2 of the zyBook. These are not optional.

Problem 6:

Read Chapter 8 from the zyBook and do all of the participation exercises therein. Which concept was most confusing for you?