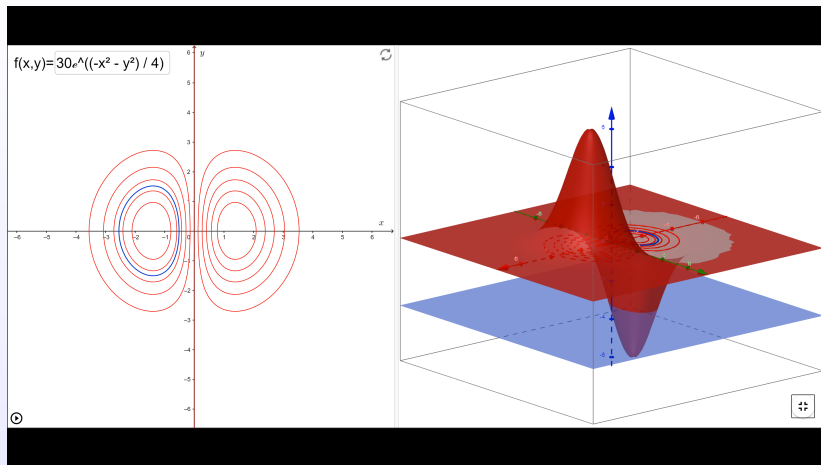


Lecture 4 (some pictures)

Math 115

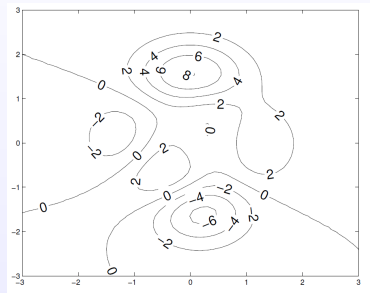
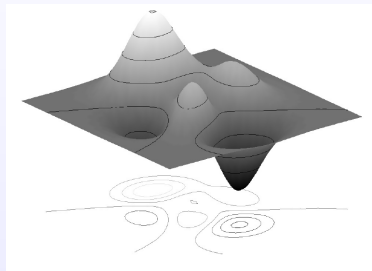
September 5, 2019

Graph and level curves

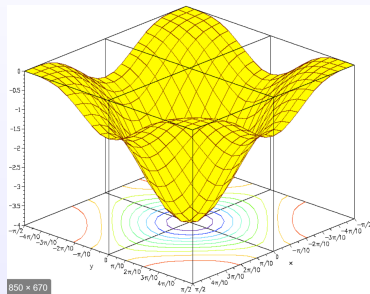
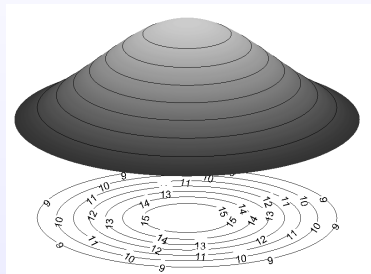


$$f(x,y) = 30e^{(-x^2 - y^2)/4} \sin(x/3)$$

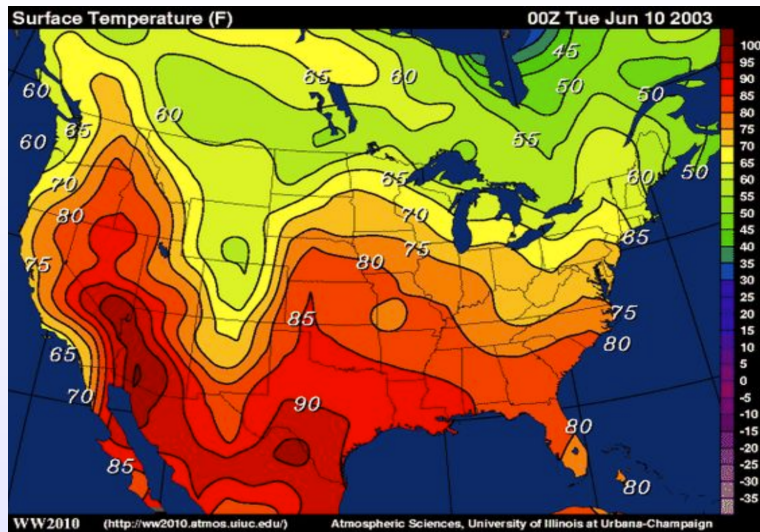
Graph and level curves



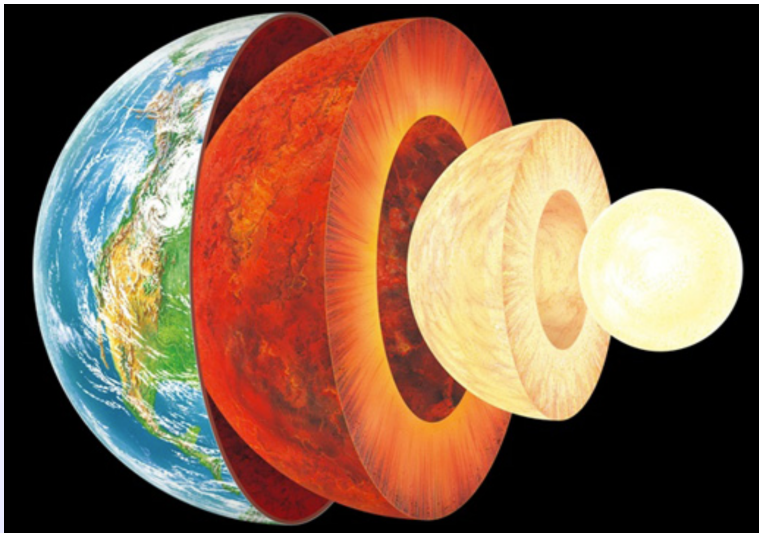
Graph and level curves



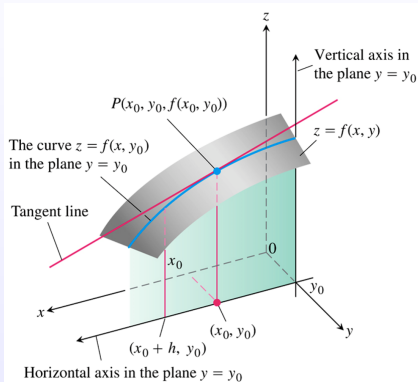
Level curves



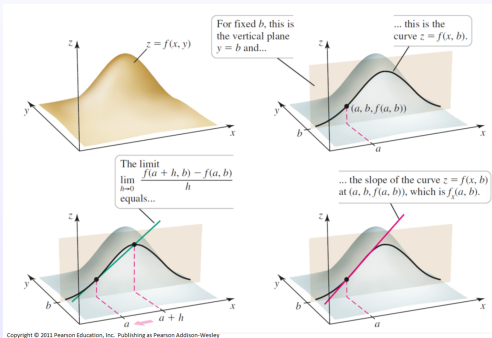
Level surfaces



Geometric interpretation of partial derivatives



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Partial derivatives: computation

Exercise: Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions:

1. $f(x, y) = 2x^2 + 4xy$

2. $f(x, y) = \sin(xy + y)$

3. $f(x, y) = f(x, y) = (1 + x^3)y^2$

4. $f(x, y) = e^x \sin y$

5. $f(x, y) = \frac{x^2}{y}$

Partial derivatives: computation

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Partial derivatives: computation

Exercise: Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions:

1. $f(x, y) = 2x^2 + 4xy \rightarrow \frac{\partial f}{\partial x}(x, y) = 4x + 4y, \quad \frac{\partial f}{\partial y}(x, y) = 4x$

Partial derivatives: computation

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2. $f(x, y) = \sin(xy + y) \rightarrow \frac{\partial f}{\partial x}(x, y) = \cos(xy + y)y,$
 $\frac{\partial f}{\partial y}(x, y) = \cos(xy + y)(x + 1)$

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 $\frac{\partial f}{\partial y}(x, y) = \cos(xy + y)(x + 1)$

3. $f(x, y) = (1 + x^3)y^2 \rightarrow \frac{\partial f}{\partial x}(x, y) = 3x^2y^2, \quad \frac{\partial f}{\partial y}(x, y) = (1 + x^3)2y$

Partial derivatives: computation

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5. $f(x, y) = \frac{x^2}{y} \rightarrow \frac{\partial f}{\partial x}(x, y) = \frac{2x}{y}, \quad \frac{\partial f}{\partial y}(x, y) = -\frac{x^2}{y^2}$

Partial derivatives: computation

Exercise: Which of the following is

$$\frac{\partial}{\partial y}(x^2 \tan y + y^2 + x)?$$

1. $\frac{2x}{\cos^2 y} + 2y + 1,$
2. $\frac{x^2}{\cos^2 y} + 2y,$
3. $2x \tan y + 1,$
4. $x^2 \tan y + y^2 + 1.$

Exercise:

$$\frac{\partial}{\partial x}(e^y \sqrt{y^2 - 2 \sin y}) = ?$$

Partial derivatives: computation

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$$\frac{\partial}{\partial y}(x^2 \tan y + y^2 + x)?$$

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2. $\frac{x^2}{\cos^2 y} + 2y,$
3. $2x \tan y + 1,$
4. $x^2 \tan y + y^2 + 1.$

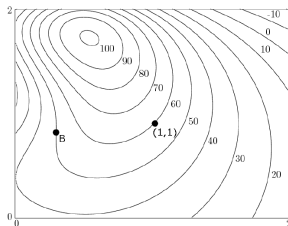
Exercise:

$$\frac{\partial}{\partial x}(e^y \sqrt{y^2 - 2 \sin y}) = 0$$

Level curves and partial derivatives

Partial Derivatives from Contour Diagrams

Example: Consider the contour diagram shown below, representing the function $h(x, y)$.



Question: $\frac{\partial h}{\partial x}(1, 1)$ is

(a) positive

(b) negative

(c) zero

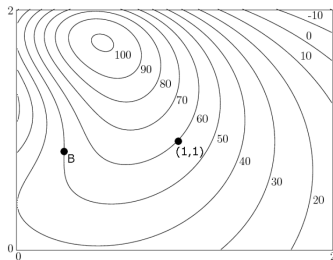
Question: $\frac{\partial h}{\partial y}(1, 1)$ is

(a) positive

(b) negative

(c) zero

Level curves and partial derivatives



Question: $\frac{\partial h}{\partial x}$ at the point B is

- (a) positive (b) negative (c) zero

Question: $\frac{\partial h}{\partial y}$ at the point B is

- (a) positive (b) negative (c) zero