# Recitation Week 2

Sections 12.2, 12.3

1. Find the limit or show that it does not exist:

   a. $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy^2-1}{y-1}$

   b. $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{x^3-y^3}{x^2+y^2}\right)$

   c. $\lim\limits_{(x,y)\to(0,0)} \dfrac{1-\cos(xy)}{xy}$ Hint: Recall Taylor polynomials.

   d. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x}{x^2+x+y^2}$

2. At what points $(x, y)$ in the plane is the following function continuous?

$$f(x,y) = \begin{cases} \dfrac{\sin(x^2+y^2)}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

3. Find all the second-order partial derivatives of $f(x, y) = x\sin(2x - y^2)$.

4. Use the limit definition of partial derivative to compute the partial derivatives of the functions at the specified points:

   - $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at $(1, 2)$ for $f(x, y) = 1 - x + xy^2$.
   - $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at $(0, 0)$ for $f(x,y) = \begin{cases} \dfrac{\sin(x^3+y^4)}{x^2+y^2} & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0). \end{cases}$