Lecture 8: Some problems

1. The Celsius temperature in a region in space is given by $T(x, y, z) = 2x^2 - xyz$. A particle is moving in this region and its position at time t is given by $x = 2t^2$, $y = 3t$, $z = -t^2$, where time is measured in seconds and distance in meters.

   - How fast is the temperature experienced by the particle changing in degrees Celsius per second at $P(8, 6, -4)$?
   - How fast is the temperature experienced by the particle changing in degrees Celsius per meter when the particle is at $P(8, 6, -4)$? Hint: In which direction is the particle moving when it is at $P(8, 6, -4)$?
   - Estimate how much the temperature will change when the particle moves 0.1 meters from $P(8, 6, -4)$.

2. Variation in electrical resistance. The resistance R produced by wiring resistors of R_1 and R_2 ohms in parallel can be calculated from the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

   - Show that

$$\Delta R \approx \left(\frac{R}{R_1}\right)^2 \Delta R_1 + \left(\frac{R}{R_2}\right)^2 \Delta R_2.$$

   - You have designed a two-resistor circuit in parallel with resistances $R_1 = 100$ ohms and $R_2 = 400$ ohms, but there is always some variation in manufacturing and the resistors by your firm will probably not have these exact values. Will the value of R be more sensitive to variation in R_1 or to variation in R_2? Give reasons for your answer.
   - In another circuit in parallel you plan to change R_1 from 20 to 20.1 ohms and R_2 from 25 to 24.9 ohms. By about what percentage will this change R?

3. Designing a soda can. A standard 12-fl-oz can of soda is essentially a cylinder of radius $r = 1$ in. and height $h = 5$ in.

   - At these dimensions, how sensitive is the can's volume to a small change in radius versus a small change in height? In other words, how much does the volume change if the radius changes by one unit? And what if the height changes by one unit?
   - Could you design a soda can that *appears* to hold more soda but in fact holds the same 12 fl oz? What might its dimensions be? (There is more than one correct answer.)