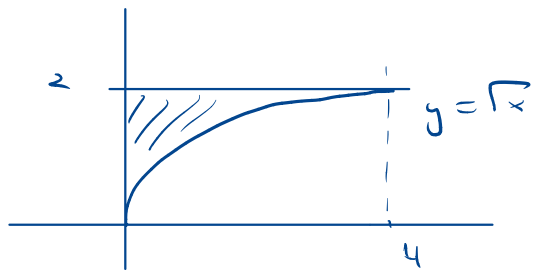
$\boxed{12}$ $\displaystyle\int_0^4 dx \int_{\sqrt{x}}^2 \frac{dy}{\sqrt{x+y^2}} = A$

Recinto: $\{(x,y)\in\mathbb{R}^2 : 0 \le x \le 4,\ \sqrt{x} \le y \le 2\}$



$$A = \int_0^2 \left(\int_0^{y^2} \frac{dx}{\sqrt{x+y^2}} \right) dy = \int_0^2 \frac{-2}{3} \frac{1}{(x+y^2)^{3/2}} \Big]_0^{y^2} dy =$$

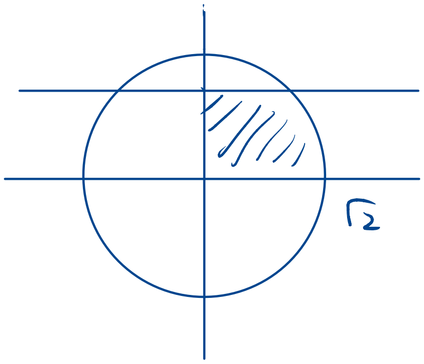
$$= \int_0^2 \frac{-2}{3}\left(\frac{1}{2^{3/2}\, y^3} - \frac{1}{y^3} \right) dy = \ldots$$

$\boxed{14}$ Volumen limitado por $x^2+y^2=1$, $x^2+y^2=4$, $z=0$,
$z = x^2+y^2$.

Cilíndricas: $1 \le r \le 2$, $0 \le z \le r^2$, $\theta \in [0, 2\pi)$

$$V = \int_0^{2\pi} \int_1^2 \left(\int_0^{r^2} r\, dz \right) dr\, d\theta .$$

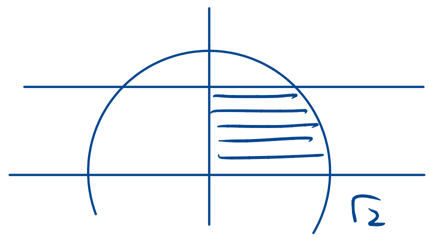
$\boxed{7}$ $A = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 2,\ 0 \leq y \leq 1,\ x \geq 0\}$



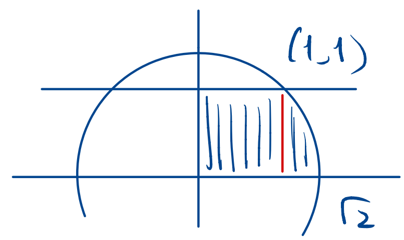
$$A = \iint_A dx\,dy$$

- <u>y-proyectable:</u> $0 \leq y \leq 1 \rightarrow x \geq 0,\ x \leq \sqrt{2-y^2}$

$$A = \int_0^1 \left(\int_0^{\sqrt{2-y^2}} dx \right) dy$$



- <u>x-proyectable:</u> dividimos en dos,



$$A = \int_0^1 \left(\int_0^1 dy \right) dx + \int_1^{\sqrt{2}} \left(\int_0^{\sqrt{2-x^2}} dy \right) dx$$

En polares: $A = \{(r,\theta) \in \mathbb{R}^2 : 0 \leq r \leq \sqrt{2},\ 0 \leq r\cdot \sin\theta \leq 1,\ \cos\theta \geq 0\}$

$$A = \int_0^{\pi/4} \left(\int_0^{\sqrt{2}} r\,dr \right) d\theta + \int_{\pi/4}^{\pi/2} \left(\int_0^{\frac{1}{\sin\theta}} r\,dr \right) d\theta$$

$$A = \int_0^1 \left(\int_0^{\pi/2} r\,d\theta \right) dr + \int_1^{\sqrt{2}} \left(\int_0^{\arcsin\left(\frac{1}{r}\right)} r\,d\theta \right) dr.$$

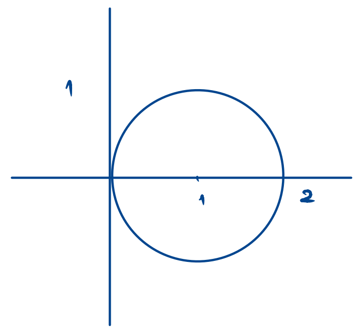
$\boxed{9}$

c) $\iint_C x \, dx \, dy$, $C = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2x\}$

$$x^2 - 2x + y^2 = (x-1)^2 + y^2 - 1 \leq 0 \Leftrightarrow (x-1)^2 + y^2 \leq 1$$

Polares:

$$\begin{cases} x = 1 + r\cos\theta \\ y = r \cdot \sin\theta \end{cases}, \quad \begin{array}{l} 0 \leq r \leq 1 \\ \theta \in [0, 2\pi) \end{array}$$



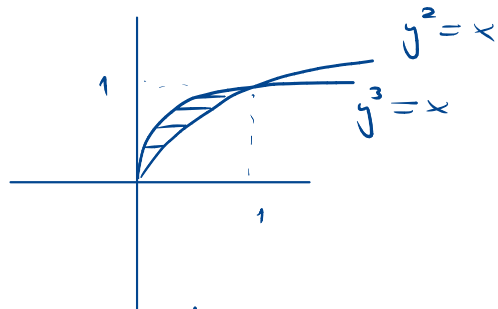
$$|J(r,\theta)| = r$$

$$\iint_C x \, dx \, dy = \iint_{\tilde{C}} (1 + r \cdot \cos\theta) r \, dr \, d\theta =$$

$$= \int_0^{2\pi} \int_0^1 (r + r^2 \cos\theta) \, dr \, d\theta = 2\pi \frac{1}{2} = \pi.$$

d) $\iint_D e^{x/y} \, dx \, dy$, $D = \{(x,y) \in \mathbb{R}^2 : y^3 \leq x \leq y^2, y \geq 0\}$

$$y^3 \leq x \leq y^2 \Rightarrow y \leq 1$$



$$\int_0^1 \left(\int_{y^3}^{y^2} e^{\frac{x}{y}} \, dx \right) dy = \int_0^1 y \, e^{\frac{x}{y}} \Big|_{y^3}^{y^2} dy = \int_0^1 \left(y e^{y} - y e^{y^2} \right) dy = \ldots$$

$\boxed{17}$ $M = \{(x,y,z) \in \mathbb{R}^3 : 1 \leq x^2+y^2 \leq 4,\ 0 \leq z \leq |x|\}$

$$V(M) = \int_0^{2\pi}\int_1^2 \left(\int_0^{r|\cos\theta|} r\,dz\right) dr\,d\theta =$$

$$= \int_0^{2\pi}\int_1^2 r^2|\cos\theta|\,dr\,d\theta = \left(\frac{8}{3}-\frac{1}{3}\right) 4\int_0^{\pi/2}\cos\theta\,d\theta = \frac{28}{3}.$$

$$\cdot \iiint_M \frac{z}{x^2+y^2}\,dx\,dy\,dz = \int_0^{2\pi}\int_1^2\left(\int_0^{r|\cos\theta|} \frac{z\cdot \cancel{r}}{r^{\cancel{2}}}\,dz\right) dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^1 \frac{1}{2} r\cos^2\theta\,dr\,d\theta = \frac{\pi}{4}.$$

$\boxed{27}$ $(a > b > c > 0)$

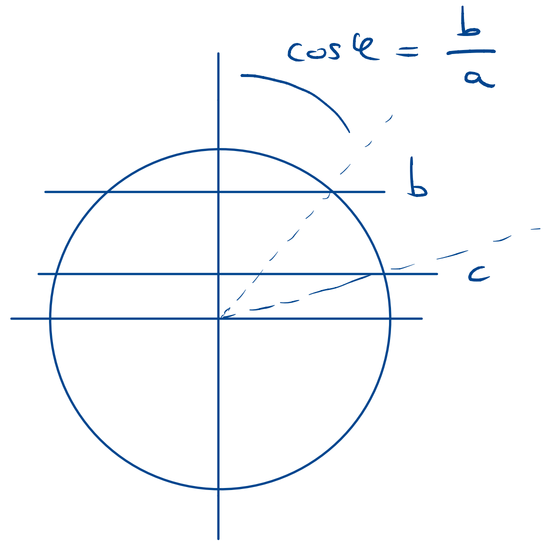
$x^2+y^2+z^2 \leq a^2,\quad z=b,\quad z=c$

(1) Coord. cilíndricas : $b \leq z \leq c$, $0 \leq r \leq \sqrt{a^2-z^2}$
$0 \leq \theta \leq 2\pi$

$$V = \int_0^{2\pi}\int_c^b\left(\int_0^{\sqrt{a^2-z^2}} r\,dr\right) dz\,d\theta = 2\pi\int_c^b \frac{1}{2}(a^2-z^2)\,dz =$$

$$= \pi\left(a^2(b-c) - \frac{1}{3}(b^3-c^3)\right).$$

② Coord. esféricas:



$c \le r \cdot \cos\varphi \le b$

$0 \le r \le a$

$\theta \in [0, 2\pi)$

$$V = \int_0^{2\pi} \int_c^b \left(\int_0^{\arccos\left(\frac{c}{r}\right)} r^2 \sin\varphi \, d\varphi \right) dr \, d\theta \; +$$

$$+ \int_0^{2\pi} \int_b^a \left(\int_{\arccos\left(\frac{b}{r}\right)}^{\arccos\left(\frac{c}{r}\right)} r^2 \sin\varphi \, d\varphi \right) dr \, d\theta =$$

$$= 2\pi \int_c^b r^2 \left(-\frac{c}{r} + 1 \right) dr + 2\pi \int_b^a r^2 \left(-\frac{c}{r} + \frac{b}{r} \right) dr =$$

$$= 2\pi \left[-c\left(\frac{b^2}{2} - \frac{c^2}{2} \right) + \frac{b^3 - c^3}{3} \right] + 2\pi \left[-c\left(\frac{a^2}{2} - \frac{b^2}{2} \right) + b\left(\frac{a^2}{2} - \frac{b^2}{2} \right) \right] =$$

$$= \pi \left(-cb^2 + c^3 + \frac{2}{3} b^3 - \frac{2}{3} c^3 + (b - c)(a^2 - b^2) \right) =$$

$$= \pi \left(a^2 (b - c) - \cancel{c} b^2 + \frac{c^3}{3} + \frac{2}{3} b^3 - b^3 + \cancel{c} b^2 \right) =$$

$$= \pi \left(a^2 (b - c) - \frac{1}{3} (b^3 - c^3) \right)$$